Abstract: The increasing number of satellites provides new opportunities. In the experiment presented in this paper, the Single Point Positioning technique is tested. Data from four different receivers were used in the tests. The GPS, Galileo and BeiDou System observations were collected over a three day long observational session. The computational process was carried out using self-made software and point positions were obtained as the result. The goal of the test was to verify the impact of the Inter-System Bias (ISB) on the final results. For this purpose, two cases of processing data were compared: with estimating ISB and without taking into account this parameter. In the paper the formulas of the mathematical models used are presented and, in both of the considered cases, a combination of GPS, BDS and Galileo was used. The results show that in the case where the ISB was taken into account, the accuracy and precision in the positioning was much better than in the approach where the ISB was not considered. Estimating the ISB allows for more precise positioning results to be obtained for car-navigation or GIS purposes.

Keywords: GPS, BDS, Galileo, SPP, ISB

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1. Introduction

For many years, there were only two operating Global Navigational Satellite Systems (GNSS) – GPS and GLONASS. However, in recent years, two new satellite systems have appeared: the Bei Dou System (BDS) and the Galileo. Galileo is predicted to be fully operational by 2020. However, it is not certain whether constellation will be ready in 2020. BDS will became fully operational in 2020. All satellites of this system are already in space, however some of them are still in their test phase. In 2012, BDS became operational above the eastern part of Asia [1–3]. Both BDS and Galileo are currently widely used in positioning [4–13]. The Single Point Positioning (SPP) is a well know method. It has been used for navigation since the 1980s. For many years, the accuracy of SPP method was limited to tens of meters, however, this has increased in recent years. This is mainly thanks to improvements such as more precise civil code C/A and more accurate ephemeris [10]. The level of 2–3 m is a current accuracy of SPP [14]. This does not change the fact that the SPP method it is still in use [11, 14, 15]. The increasing number of GNSS satellites has the potential to improve the accuracy of the SPP method.

2. GPS, BDS and Galileo Systems Description

The GPS System is an American navigational system created during “Cold War”. The system became fully operational in 1993 with 24 satellites. Nowadays, GPS operates with so called “Expandable 24”, which means that the full constellation consists of 27 satellites [2, 16]. However, most of the time over 30 GPS satellites are available for users. The GPS satellites are placed on six orbital planes at an altitude of approximately 20,200 km [17, 18]. The inclination for the GPS satellites planes is 55° [17]. The GPS system uses the following frequencies: L1, L2 and L5 [11]. The frequencies in megahertz [MHz] are shown in Table 1 at the end of Chapter 2. The GPS consists of three segments: the space segment which has already been described above; the control segment which consists of a global network of ground facilities that track the GPS satellites, monitors their transmissions, performs analyses, and sends commands and data to the constellation a third segment which is the user segment. Currently, the number of GPS satellites oscillates between 30 and 32.

The BeiDou System (BDS) is a Chinese navigational system and has been operational above the East Asia region since December 27 2012. The BDS full constellation will consist of 35 satellites: 5 geostationary satellites (GEO), 3 Inclined Geosynchronous Orbits (IGSO) and 27 medium-Earth orbits (MEO) [19, 20]. All of these satellites are currently in space. For the IGSO and GEO satellites the altitude is approximately 35,786 km when for the MEO satellites the approximated altitude is 21,528 km. For both MEO and IGSO the inclination is 55° [3]. For the MEO orbit planes period is...
The China Geodetic Coordinate System 2000 (CGCS2000) is used as a reference frame and BeiDou Time (BDT) is used as a time system. January 6, 1980, 00:00:00, UTC is an initial epoch [21]. Before 2015, BDS used the following frequencies: B1, B2 and B3. After modernization, so after 2015, BDS use: B1-3, B2a, B2b. The B3 frequency is the same as before modernization [19, 22]. The frequencies are presented in Table 1. Figure 1 presents the BDS satellites footprints. The BDS system also consist of three segments, space, ground and user segments. There are 33 BeiDou System 3 satellites in space, and 16 BDS-2 satellites. Some of the BDS-3 satellites are still in the testing phase.

The European navigation system is called Galileo, and just like BDS is a civil system unlike GPS or GLONASS. Galileo is expected to be fully operational in 2020. The Galileo satellites are placed on the three circular medium Earth orbits at an altitude of 23,222. The Galileo full constellation will consist of 30 satellites. 24 of those satellites will be operational and 6 satellites will be spare [7]. The inclination of the Galileo system orbits is 56° [1]. Galileo System Time (GST) is used a time system for Galileo, 22 August 1999 00:00:00 UTC is the start epoch of GST. The Galileo Terrestrial Reference Frame (GTRF) is the reference frame for Galileo [2, 17]. Galileo uses the following signal frequencies: E1, E5a, E5b, E5 and E6 [2]. The frequencies are also shown in Table 1. Figure 2 shows the Galileo footprints. There are 26 Galileo satellites placed on their orbits. Some of them are still in their test phase and will soon be included in the Galileo constellation.

Table 1 shows the frequencies of GPS, Galileo and BDS [19, 22, 23].
Table 1. BDS, GPS and Galileo frequencies and signals

<table>
<thead>
<tr>
<th>Signal</th>
<th>GPS Frequency [MHz]</th>
<th>Signal</th>
<th>Galileo Frequency [MHz]</th>
<th>Signal</th>
<th>BDS Frequency [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>1575.42</td>
<td>E1</td>
<td>1575.42</td>
<td>B1-3</td>
<td>1575.42</td>
</tr>
<tr>
<td>L2</td>
<td>1227.60</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<tr>
<td>L5</td>
<td>1176.45</td>
<td>E5a</td>
<td>1176.45</td>
<td>B2a</td>
<td>1176.45</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>E5b</td>
<td>1207.14</td>
<td>B2b</td>
<td>1207.140</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>E5</td>
<td>1191.795</td>
<td>B2-3</td>
<td>1191.795</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>E6</td>
<td>1278.75</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>B1</td>
<td>1561.098</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>B2</td>
<td>1207.14</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>B3</td>
<td>1268.52</td>
</tr>
</tbody>
</table>

Source: own study based on [22, 23]

3. SPP and Inter-System Bias

The pseudo-range between a receiver and a satellite determined by the signal travel time is used in the Single Point Positioning (SPP) method. To obtain the pseudo-range, the following Equation (1) can be used [15, 24]:

\[
D = \rho - (ISB + c\delta t_u) + c\hat{t}_s + d_e + d_i + \varepsilon_p
\]  
(1)
where:

- \( D \) – pseudorange,
- \( \rho \) – geometric distance between satellite and receiver,
- \( c\delta t_s \) – satellite clock offset scaled by the speed of light,
- \( c\delta t_u \) – receiver clock offset scaled by the speed of light,
- \( d_t \) – tropospheric delay,
- \( d_i \) – ionospheric delay,
- \( \varepsilon_o \) – other unmodelled errors.

The \( c\delta t_s, d_t, d_i \) are computed and incorporated into \( D \) term. The \( c\delta t_u \) is included in ISB (Inter-System bias) term.

The ISBs_{sys} (merged with receiver clock offset) and the three-dimensional coordinates of receiver is estimated \([15]\). To create a design matrix \( A \) in the part of experiment where the Inter-System Bias was taken into account, we use Equation \((2)\):

\[
A_{ISB} = \begin{bmatrix}
\frac{x^{G1} - x_{A0}}{\rho_{G1}^{A0}} & \frac{y^{G1} - y_{A0}}{\rho_{G1}^{A0}} & \frac{z^{G1} - z_{A0}}{\rho_{G1}^{A0}} & 1 & 0 & 0 \\
\frac{x^{G2} - x_{A0}}{\rho_{G2}^{A0}} & \frac{y^{G2} - y_{A0}}{\rho_{G2}^{A0}} & \frac{z^{G2} - z_{A0}}{\rho_{G2}^{A0}} & 1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x^{Gnf} - x_{A0}}{\rho_{Gnf}^{A0}} & \frac{y^{Gnf} - y_{A0}}{\rho_{Gnf}^{A0}} & \frac{z^{Gnf} - z_{A0}}{\rho_{Gnf}^{A0}} & 1 & 0 & 0 \\
\frac{x^{E1} - x_{A0}}{\rho_{E1}^{A0}} & \frac{y^{E1} - y_{A0}}{\rho_{E1}^{A0}} & \frac{z^{E1} - z_{A0}}{\rho_{E1}^{A0}} & 0 & 1 & 0 \\
\frac{x^{E2} - x_{A0}}{\rho_{E2}^{A0}} & \frac{y^{E2} - y_{A0}}{\rho_{E2}^{A0}} & \frac{z^{E2} - z_{A0}}{\rho_{E2}^{A0}} & 0 & 1 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x^{Enf} - x_{A0}}{\rho_{Enf}^{A0}} & \frac{y^{Enf} - y_{A0}}{\rho_{Enf}^{A0}} & \frac{z^{Enf} - z_{A0}}{\rho_{Enf}^{A0}} & 0 & 1 & 0 \\
\frac{x^{C1} - x_{A0}}{\rho_{C1}^{A0}} & \frac{y^{C1} - y_{A0}}{\rho_{C1}^{A0}} & \frac{z^{C1} - z_{A0}}{\rho_{C1}^{A0}} & 0 & 0 & 1 \\
\frac{x^{C2} - x_{A0}}{\rho_{C2}^{A0}} & \frac{y^{C2} - y_{A0}}{\rho_{C2}^{A0}} & \frac{z^{C2} - z_{A0}}{\rho_{C2}^{A0}} & 0 & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x^{Cnf} - x_{A0}}{\rho_{Cnf}^{A0}} & \frac{y^{Cnf} - y_{A0}}{\rho_{Cnf}^{A0}} & \frac{z^{Cnf} - z_{A0}}{\rho_{Cnf}^{A0}} & 0 & 0 & 1
\end{bmatrix}
\]
When the ISB was not taken into account, the matrix was created as follows:

\[
A_{noISB} = \begin{bmatrix}
\frac{x^{G1} - x_{A0}}{\rho^{G1}_{A0}} & \frac{y^{G1} - y_{A0}}{\rho^{G1}_{A0}} & \frac{z^{G1} - z_{A0}}{\rho^{G1}_{A0}} & 1 \\
\frac{x^{G2} - x_{A0}}{\rho^{G2}_{A0}} & \frac{y^{G2} - y_{A0}}{\rho^{G2}_{A0}} & \frac{z^{G2} - z_{A0}}{\rho^{G2}_{A0}} & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\frac{x^{Gn^G} - x_{A0}}{\rho^{Gn^G}_{A0}} & \frac{y^{Gn^G} - y_{A0}}{\rho^{Gn^G}_{A0}} & \frac{z^{Gn^G} - z_{A0}}{\rho^{Gn^G}_{A0}} & 1 \\
\frac{x^{E1} - x_{A0}}{\rho^{E1}_{A0}} & \frac{y^{E1} - y_{A0}}{\rho^{E1}_{A0}} & \frac{z^{E1} - z_{A0}}{\rho^{E1}_{A0}} & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\frac{x^{En^E} - x_{A0}}{\rho^{En^E}_{A0}} & \frac{y^{En^E} - y_{A0}}{\rho^{En^E}_{A0}} & \frac{z^{En^E} - z_{A0}}{\rho^{En^E}_{A0}} & 1 \\
\frac{x^{C1} - x_{A0}}{\rho^{C1}_{A0}} & \frac{y^{C1} - y_{A0}}{\rho^{C1}_{A0}} & \frac{z^{C1} - z_{A0}}{\rho^{C1}_{A0}} & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\frac{x^{Cn^C} - x_{A0}}{\rho^{Cn^C}_{A0}} & \frac{y^{Cn^C} - y_{A0}}{\rho^{Cn^C}_{A0}} & \frac{z^{Cn^C} - z_{A0}}{\rho^{Cn^C}_{A0}} & 1 \\
\end{bmatrix}
\]

where in (2) and (3): \(xyz^G\), \(xyz^E\), \(xyz^C\) are BDS, Galileo and GPS satellites coordinates, \(\rho^{G\nu}_{A0}\), \(\rho^{E\nu}_{A0}\), \(\rho^{C\nu}_{A0}\) are geometric distances between approximate point position and satellites, \(n\) stands for the number of observable satellites, \(xyz_{A0}\) are approximate point coordinates. Columns four, five and six in the Equation (2) are used to estimate receiver clock error and Inter-System bias. In the following approach, the receiver clock is absorbed by code biases during the parameter estimation process [11]. During the de-correlation process, the bias for code observations may occur. The ISB should be taken into the account during the positioning with different GNSS systems. In the case of Equation (3), the ISB is not estimated. The presented design matrix \(A\) is used in absolute SPP positioning where the least squares estimation method is used

\[
X = (A^T PA)^{-1} (A^T PL)
\]
In Equation (4) $P$ is a weight matrix presented in further part of paper, $L$ is a free terms matrix and $A$ is a design matrix. By solving this equation (4) we get increments to coordinates $\Delta x$, $\Delta y$ and $\Delta z$ and a clock error $\Delta t$. Matrix $X$ looks like follows:

$$X = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta t \end{bmatrix}$$

(5)

4. Experiment Description

In the experiment, data from four GNSS stations were used with three different receivers (two stations WTZ2 and WROC use the same receivers). The JFNG station was equipped with the TRIMBLE NETR9 5.22 receiver, the WROC station use the LEICA GR50 4.11/7.102 receiver. The same receiver is used by the WTZ2 station. The NNOR station uses the SEPT POLARX4 2.9.5-extref1 receiver. The data was obtained at 5 minute intervals. GPS P1 observations were used, for Galileo E1 observations were used and for BDS B1 observations were used. The ionospheric and tropospheric corrections were modelled. For tropospheric corrections, the Vienna Mapping Function (VMF1) was used [25]. The Technical University of Catalonia ionosphere maps were used for ionosphere correction. The precise orbits in sp3 format were obtained and used during the tests. In the experiment the two different approaches were compared. In the first one, the Intersystem Bias was not taken into the account, and the one receiver clock offset was estimated for all three GNSS systems: GPS, BDS and Galileo. In the second approach, the parameter which includes receiver clock offset and ISB was estimated for each system separately. The values of this parameter were compared. The results of the SPP between these two approaches were also compared to see whether the estimation of ISB has an influence on the received position. To determine the accuracy of positioning, the obtained SPP positions were compared to GNSS stations coordinates. The obtained biases were transformed into a local topocentric coordinate system and presented. The weight matrix $P$ was created as follows:

$$P = \begin{bmatrix} \frac{\sin(El_1)^2}{\sigma^2} & 0 & 0 & 0 \\ 0 & \frac{\sin(El_2)^2}{\sigma^2} & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \frac{\sin(El_n)^2}{\sigma^2} \end{bmatrix}$$

(6)
where $E_l$ is a satellite elevation of each satellite in given epoch, $\sigma$ stands for a code observation standard deviation (STD). For BDS and GPS, the STD was set to 0.6 m and 0.3 m for Galileo [10]. Elevation mask was set to the $5^\circ$.

All tests were performed using self-made software in MatLab. Software reads RINEX 2 and RINEX 3, sp3 files with precise clocks and satellites orbits, as well as other necessary data. All the computations were performed using self-made software.

5. Experiment Results

The Figure 3 shows the number of satellites available during the experiment for each station. It is visible that the number of available BDS satellites is higher in the NNOR and JFNG stations than in the case of GPS when on stations WROC and WTZ2 number of BDS satellites is comparable with number of Galileo satellites, a result of the BDS constellation characteristics. The common number of satellites for WROC and WTZ2 stations is about 15 when for stations JFNG and WROC number of common satellites hovers around 20. Figure 4 shows the TDOP factor values observed on each station. It can be observed that on each station, the TDOP factor for Galileo is the highest. It was probably caused by the low number of available satellites for this system. For conjoined clocks, the value of TDOP is the lowest.
Figure 5 shows the results of the receivers clock offset estimation. For each receiver a correlation between offsets for GPS, Galileo and BDS is noticeable. The differences between each estimated offset are the Inter-System bias and the differences between systems are not constant because of noises. The receiver clock offset estimated jointly has noticeable correlation with the results obtained for each system separately. However, it behaves less stably that the offset estimated as parameter including ISB. For the LEICA GR50 4.11/7.102 receivers the values of estimated receiver clock offsets are different, however it is noticeable that they act similarly. There is no visible correlation between number of satellites and the estimated clock error.
However, we can observe a correlation between TDOP factor values and estimated clock offsets. When the TDOP factor values increase, we can observe significant changes in the estimated offset values, regardless of the GNSS system.

Table 2 shows the results of the SPP positioning when estimating ISB and without it. It is clearly visible that results obtained with estimated ISB are more precise and accurate. This is especially visible for the up direction in the local topocentric coordinate system, where the obtained mean and standard deviation (STD) values are much lower that for the approach where the ISB is not taken into account. The vertical mean results seem to be similarly accurate, but the STD values show that results when ISB is taken into account are more accurate.

Fig. 5. Estimated receivers clock offsets
Table 2. Single Point Positioning results

<table>
<thead>
<tr>
<th></th>
<th>JFNG</th>
<th>WROC</th>
<th>WTZ2</th>
<th>NNOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>dN</td>
<td>0.36</td>
<td>1.25</td>
<td>1.20</td>
<td>0.42</td>
</tr>
<tr>
<td>Std</td>
<td>1.83</td>
<td>2.10</td>
<td>2.29</td>
<td>1.69</td>
</tr>
<tr>
<td>dE</td>
<td>1.70</td>
<td>1.54</td>
<td>0.35</td>
<td>2.20</td>
</tr>
<tr>
<td>Std</td>
<td>1.05</td>
<td>1.26</td>
<td>1.23</td>
<td>1.14</td>
</tr>
<tr>
<td>dU</td>
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<td>-100</td>
<td>-1.79</td>
<td>0.04</td>
</tr>
<tr>
<td>Std</td>
<td>2.97</td>
<td>4.91</td>
<td>4.94</td>
<td>3.68</td>
</tr>
</tbody>
</table>

ISB not taken into account

<table>
<thead>
<tr>
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<th>JFNG</th>
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<th>WTZ2</th>
<th>NNOR</th>
</tr>
</thead>
<tbody>
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<td>dN</td>
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<tr>
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<td>Std</td>
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<td>7.70</td>
<td>7.43</td>
<td>6.51</td>
</tr>
</tbody>
</table>

Source: own study based on MatLab results

6. Summary and Conclusions

In this study, the need to take into account the Inter-System Bias was presented and evaluated. The experiment used three day long data from 4 GNSS stations: WTZ2, WROC (equipped with the same type of the receiver), JFNG and NNOR. The results from each station are presented.

By taking into account Inter-System Bias, both positioning accuracy and availability are improved. For Single Point Positioning, the obtained results are more accurate and precise when the receiver clock offsets including ISB were estimated for each GPS, Galileo and BDS separately that when there was only one receiver clock offset estimated for all three systems. In the experiment it was also shown that there is a correlation between estimated clocks for GPS, Galileo and BDS separately and the difference in the values is an Inter-System Bias. Estimating ISB allows for more precise positioning results to be obtained for car-navigation or GIS purposes.

Acknowledgment

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References


